

# What's in the Bag?

Lecture 4

Sections 1.4.1-1.4.2

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# Outline

- 1 Bag A and Bag B
- 2 Possible Decision Rules
  - Decision Rule #1
  - Decision Rule #2
  - Decision Rule #3
- 3  $\alpha$  vs.  $\beta$
- 4 Assignment

## Example (Review Quiz)

- 1 The results of a study were found not to be statistically significant. We may conclude that
- (a) The null hypothesis was accepted.
  - (b) The null hypothesis was not accepted.
  - (c) The alternative hypothesis was accepted.
  - (d) We cannot tell which was accepted.

## Example (Review Quiz)

- 2 The results of a study were found not to be statistically significant. We may conclude that
- (a) A Type I error could have been made.
  - (b) A Type II error could have been made.
  - (c) Either type of error could have been made.
  - (d) Neither type of error could have been made.

# Review Quiz Answers

## Example (Review Quiz Answers)

1. (a) The null hypothesis was accepted.
2. (b) A Type II error could have been made.

# Outline

## 1 Bag A and Bag B

## 2 Possible Decision Rules

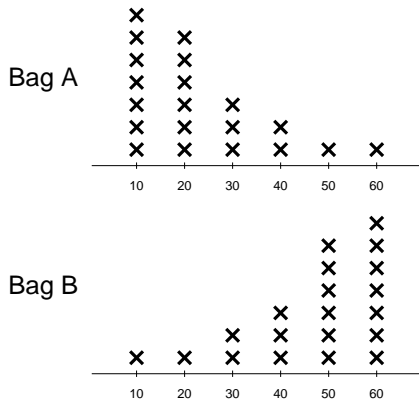
- Decision Rule #1
- Decision Rule #2
- Decision Rule #3

## 3 $\alpha$ vs. $\beta$

## 4 Assignment

# Two Bags

- Two bags contain vouchers worth various dollar amounts.



# Two Bags

- The vouchers in Bag A are worth \$470.
- The vouchers in Bag B are worth \$930.
- You get to cash in the vouchers in whichever bag you want.
- Which bag should you choose?

# Two Bags

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- The vouchers in Bag B are worth \$930.
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- Which bag should you choose?
- Oh, one more thing...

# Two Bags

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- The vouchers in Bag B are worth \$930.
- You get to cash in the vouchers in whichever bag you want.
- Which bag should you choose?
- Oh, one more thing...
- You are handed one of the bags, but you do not know which bag is it.

# Two Bags

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- The vouchers in Bag B are worth \$930.
- You get to cash in the vouchers in whichever bag you want.
- Which bag should you choose?
- Oh, one more thing...
- You are handed one of the bags, but you do not know which bag is it.
- You may look at one voucher in that bag.

# Two Bags

- The vouchers in Bag A are worth \$470.
- The vouchers in Bag B are worth \$930.
- You get to cash in the vouchers in whichever bag you want.
- Which bag should you choose?
- Oh, one more thing...
- You are handed one of the bags, but you do not know which bag is it.
- You may look at one voucher in that bag.
- Based on that alone, you must choose which bag to cash in.

# The Hypotheses

- The hypotheses are:
  - $H_0$ : You are holding Bag A.
  - $H_1$ : You are holding Bag B.
- You will choose one voucher at random from the bag and read its value.
- From that information, you will decide which bag you believe you are holding.
- Then you will choose the one that you believe is Bag B, take the money and run.

# Decisions and Their Consequences

		The State of Nature	
		It is Bag A	It is Bag B
Our Decision	It is Bag A (switch bags)	Win \$930	Win \$470
	It is Bag B (keep the bag)	Win \$470	Win \$930

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3  $\alpha$  vs.  $\beta$

4 Assignment

# Decision Rules

- There are several possible decision rules, some better than others.
- We will look at three of them.

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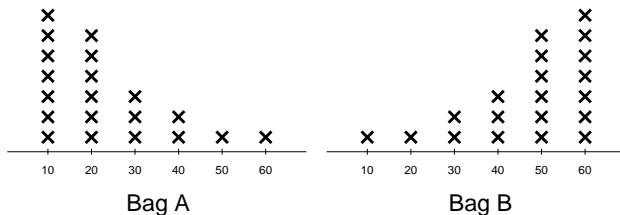
4 Assignment

# A Decision Rule

## Decision Rule #1

Reject  $H_0$  if the voucher is worth \$60.

- What is  $\alpha$ ?
- What is  $\beta$ ?



# Rejection and Acceptance Regions

## Definition (Rejection Region)

The **rejection region** is the set of possible observed values that will lead to rejection of the null hypothesis.

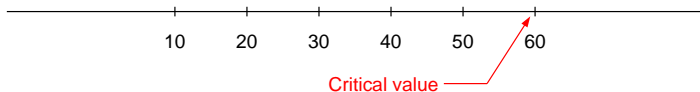
## Definition (Acceptance Region)

The **acceptance region** is the set of possible observed values that will lead to acceptance of the null hypothesis.

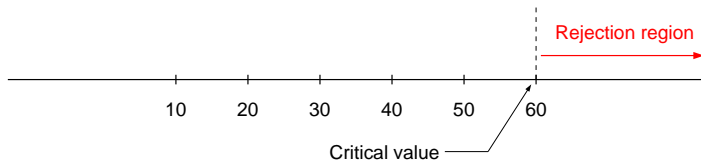
## Definition (Critical Value(s))

The **critical value(s)** is the value that separates the the two regions. The critical value itself will be included in the rejection region.

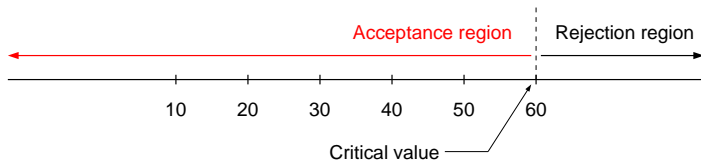
# Decision Rule #1



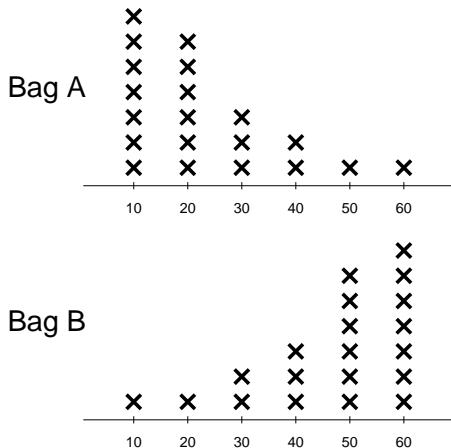
# Decision Rule #1



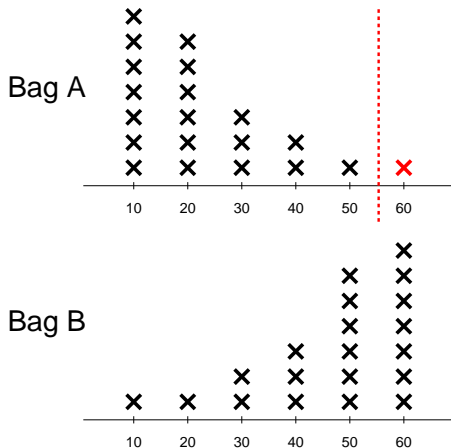
# Decision Rule #1



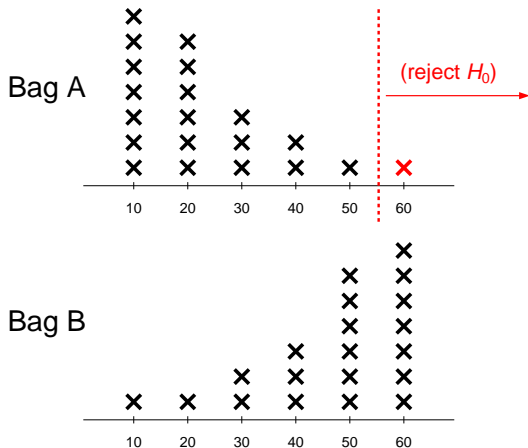
# Compute $\alpha$



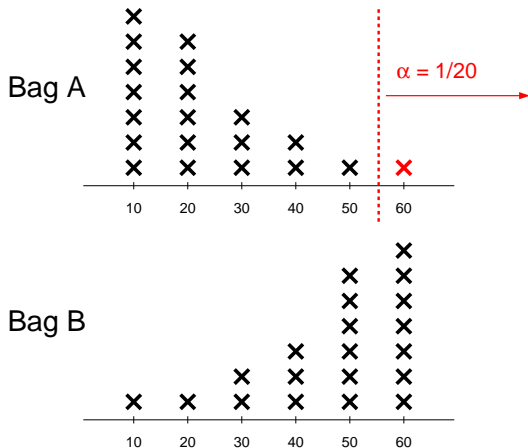
# Compute $\alpha$



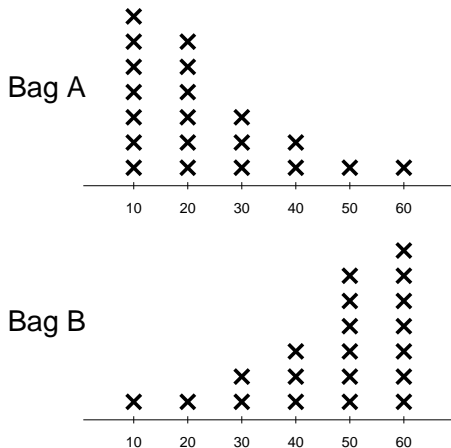
# Compute $\alpha$



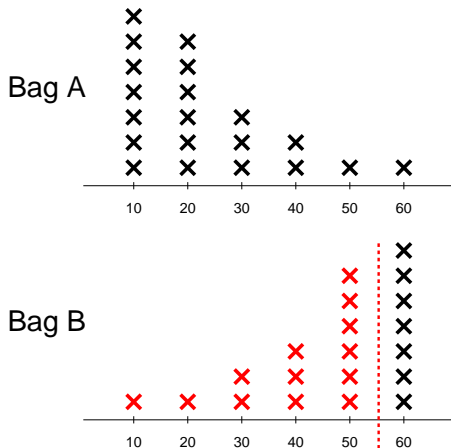
# Compute $\alpha$



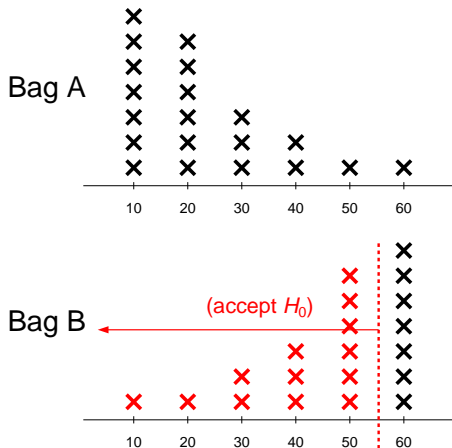
# Compute $\beta$



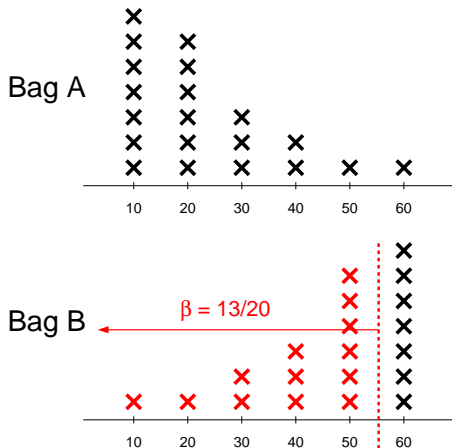
# Compute $\beta$



# Compute $\beta$



# Compute $\beta$



## Definition (Direction of Extreme)

The **direction of extreme** is the direction, left or right or both, in which we are more likely to reject  $H_0$ .

- Normally, either
  - The larger the observed value, the more likely we are to reject  $H_0$ ,  
or
  - The smaller the observed value, the more likely we are to reject  $H_0$ .

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4 Assignment

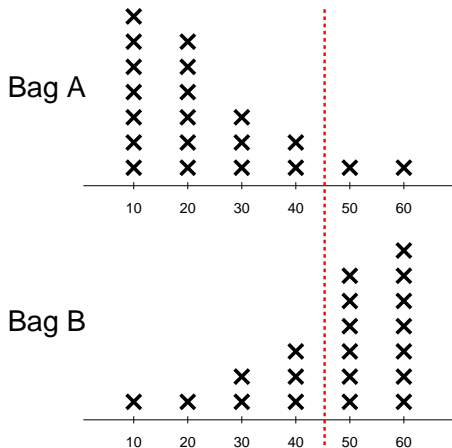
# Another Decision Rule

## Decision Rule #2

Reject  $H_0$  if the voucher is worth at least \$50.

- What is  $\alpha$ ?
- What is  $\beta$ ?

# Compute $\alpha$ and $\beta$



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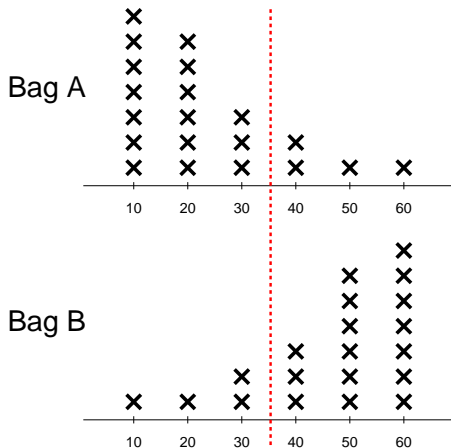
# Yet Another Decision Rule

## Decision Rule #3

Reject  $H_0$  if the voucher is worth at least \$40.

- What is  $\alpha$ ?
- What is  $\beta$ ?

# Compute $\alpha$ and $\beta$



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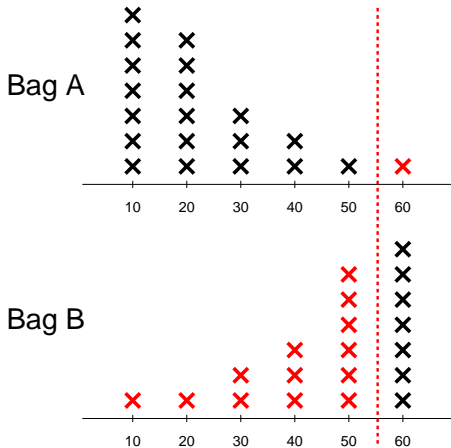
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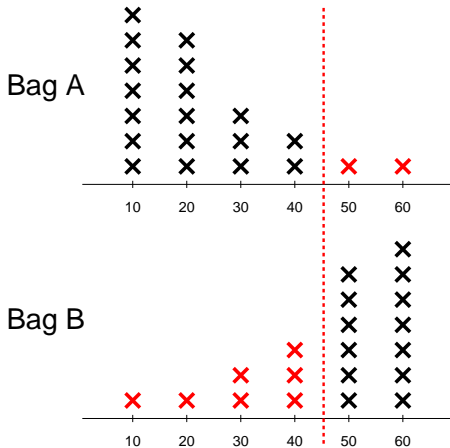
# $\alpha$ vs. $\beta$

- If we modify the decision rule to decrease  $\alpha$ , we will increase  $\beta$ , and
- If we modify the decision rule to decrease  $\beta$ , we will increase  $\alpha$ .

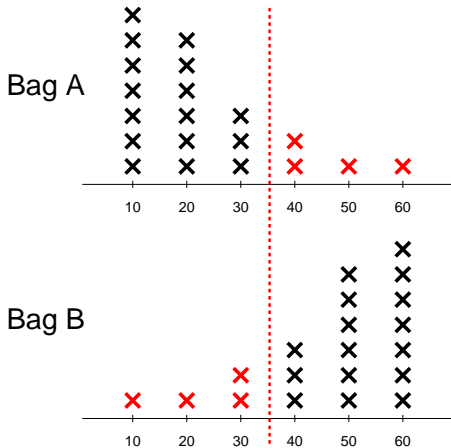
# $\alpha$ vs. $\beta$



# $\alpha$ vs. $\beta$



# $\alpha$ vs. $\beta$



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## Homework

- Read Section 1.4 - 1.4.2, pages 16 - 28.
- Let's Do It! 1.7, 1.8.
- Page 67, exercises 13 - 15, 17 - 23, 25.